Synthèse

Binômes

1) mise en évidence

2) produit remarquable : $a^2 - b^2 = (a + b)(a - b)$

Trinômes

1) mise en évidence

2) produits remarquables : $a^2 + 2ab + b^2 = (a + b)^2$

$$a^2 - 2ab + b^2 = (a - b)^2 = (b - a)^2$$

3) discriminant : $ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadri nômes (et +)

1) mise en évidence

2) produits remarquables

3) discriminant

Remarques : il faut toujours factoriser au maximum !
$a^2 + b^2$: pas moyen de factoriser

Exercices

1. **Factorise à l'aide de la mise en évidence**

1) $15a^7b^2 - 10a^5b^3 = 5a^5b^2 (3a^2 - 2b)$

2) $12x^2y^2 - 18xy^3 + 24x^3y = 6xy (2y - 3y^2 + 4x^2)$

3) $(x + 1)^2 - 3(x + 1) = (x + 1) (x + 1 - 3) = (x + 1) (x - 2)$

4) $2(n + 1) n + 3(n + 1) = (n + 1) (2n + 3)$

5) $a(x - y) - (y - x)^2 = a(x - y) - (x - y)^2 = (x - y) [a - (x - y)] = (x - y) (a - x + y)$

6) $(x + 2)(x - 3) - (x - 3) (x + 1) = (x - 3) [(x + 2) - (x + 1)]$
$= (x - 3) (x + 2 - x - 1) = (x - 3) 1 = (x - 3)$

7) $3(x - 2) x - (x + 2)^2 + (x + 2) = 3x(x - 2) + (x + 2) [-(x + 2) + 1)]$
$= 3x(x - 2) + (x + 2) (-x - 2 + 1)$
$= 3x(x - 2) + (x + 2) (-x - 1)$

8) $35x^3y^2 - 49x^2y^3 = 7x^2y^2 (5x - 7y)$

9) $3x(x + y)^3 - 4x^2(x + y)^2 = x(x + y)^2 [3 (x + y) - 4x] = x(x + y)^2 (3x + 3y - 4x)$
$= x(x + y)^2 (-x + 3y)$

10) $y (b - a) + x (a - b) = y (b - a) - x (-a + b) = y (b - a) - x (b - a) = (y - x) (b - a)$

11) $5x(x + 1)^2 - 10x^2(x + 1) = (x + 1) [5x (x + 1) - 10x^2] = (x + 1) (5x^2 + 5x - 10x^2)$
$= (x + 1) (-5x^2 + 5x) = 5x (x + 1) (-x + 1)$

12) $6x^2 (u - 1) - 4x (1 - u) = 2x (u - 1) (3x - 2)$

13) $x (2a - b) + (b - 2a) = x (2a - b) - (-b + 2a) = x (2a - b) - (2a - b) = (2a - b) (x - 1)$

2. Même exercice

1) $ab - b^2 =$ **$b (a - b)$**

2) $x\,y + y =$ **$y (x + 1)$**

3) $a^2b - ab^2 =$ **$ab (a - b)$**

4) $x\,y - 2y =$ **$y (x - 2)$**

5) $x^3y^2 - x^2y^3 =$ **$x^2y^2 (x - y)$**

6) $6x^2y + 4xy^2 =$ **$2xy (3x + 2y)$**

7) $15x^7b^2 - 10x^5b^3 =$ **$5x^5b^2 (3x^2 - 2b)$**

8) $y (b - a) + b (b - a) =$ **$(b - a) (y + b)$**

9) $3(x + y) - 2(x - y) =$ **$3x + 3y - 2x + 2y = x + 5y$**

10) $12a^2b^2 - 18ab^3 + 24a^3b =$ **$6ab (2ab - 3b^2 + 4a^2)$**

11) $12x^2y^3 - 30x^3y^2 + 18xy^4 =$ **$6xy^2 (2xy - 5x^2 + 3y^2)$**

12) $3x^2 - 9xy^2 + 6x =$ **$3x (x - 3y^2 + 6)$**

13) $3xyz^3 - 21 x^2y^2z^2 - 6x^3y^3z =$ **$3xyz (z^2 - 7xyz - 2x^2y^2)$**

14) $5a (a + 2)^2 - 3a^2 (a + 2) =$ **$a (a + 2) [(5 (a + 2) - 3a] = a (a + 2) (5a + 10 - 3a)$**
   $= a (a + 2) (2a + 10) = 2a (a + 2) (a + 5)$

15) $5(x + y - z) - 10x(x + y - z) =$ **$(x + y - z) (5 - 10x) = 5(x + y - z) (1 - 2x)$**

16) $45x^3y^4z^5 + 60x^5y^2z - 90x^4y^3z^2 =$ **$15x^3y^2z (3y^2z^4 + 4x^2 - 6xyz)$**

17) $39a^5b^5c^3 - 65a^5b^3cd =$ **$a^5b^3c\ (39b^2c^2 - 65d)$**

18) $-51a^4b^5c + 17a^3b^2c - 32a^3b^5c^4 =$ **$a^3b^2c\ (-51ab^3 + 17 - 32b^3c^3)$**

19) $a(x - y) + b(x - y) =$ **$(x - y)\ (a + b)$**

20) $2a\ (b - c) - 4c\ (b - c) =$ **$2(b - c)\ (a - 2c)$**

21) $9a^2b(x + y) + 3ab^2(x + y) =$ **$3ab(x + y)\ (3a + b)$**

22) $a\ (m - n) - b\ (n - m) =$ **$a\ (m - n) + b\ (m - n) = (m - n)\ (a + b)$**

23) $5a^2\ (b - 2) + 15a\ (2 - b) =$ **$5a\ [a\ (b - 2) - 3(b - 2)] = 5a\ (b - 2)\ (a - 3)$**

3. Factorise à l'aide des produits remarquables

1) $1/9 - x^2 =$ **$(1/3 - x)\ (1/3 + x)$**

2) $x^2 - 144 =$ **$(x - 12)\ (x + 12)$**

3) $a^2 - 4a - 4 =$ **$(a + 4,87)\ (a - 0,85)$**   **Ce n'est pas un produit remarquable.**

On peut utiliser la méthode du discriminant $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = \dfrac{-4 \pm \sqrt{(-4)^2 - 4.1.(-4)}}{2.1} = \dfrac{-4 \pm \sqrt{(16 + 16)}}{2} = \dfrac{-4 \pm \sqrt{32}}{2} = \dfrac{-4 \pm 5,7}{2} = \dfrac{-4 \pm 5,7}{2}$ ⟹ $x_1 = \dfrac{-4 + 5,7}{2} = \dfrac{1,7}{2} = 0,85$

$x_2 = \dfrac{-4 - 5,7}{2} = \dfrac{-9,7}{2} = -4,87$

4) $x^2 - 8x + 16 =$ **$(x - 4)^2$**

5) $25x^2 + 30x + 9 =$ **$(5x + 3)^2$**

6) $x^2 - \dfrac{2x}{6} + \dfrac{1}{9} =$ **$(x - \dfrac{1}{3})^2$**

7) $49 - 14a + a^2 =$ **$(7 - a)^2$**

8) $9a^2 - 12ab + 4b^2 =$ **$(3a - 2b)^2$**

9) $x^2 + 4xy + 4y^2 = (x + 2y)^2$

10) $\dfrac{a^2}{9} + \dfrac{2ab}{15} + \dfrac{b^2}{25} = (\dfrac{a}{3} + \dfrac{b}{5})^2$

11) $16x^2 - 4 = 4(4x^2 - 1) = 4(x - 1)(x + 1)$

12) $a^4 - 81 = (a^2 - 9)(a^2 + 9) = (a - 3)(a + 3)(a^2 + 9)$

13) $1 + 2x^2 + x^4 = (1 + x^2)^2$

14) $x^2 - 4xy + 4y^2 = (x - 2y)^2$

15) $x^6 - 6x^3 + 9 = (x^3 - 3)^2$

16) $x^4 - 18x^2 + 81 = (x^2 - 9)^2 = [(x - 3)(x + 3)]^2 = (x - 3)^2(x + 3)^2$

17) $4x^2 - 28x + 49 = (2x - 7)^2$

18) $xy^2 - y^2z = y^2(x - z)$ **C'est par la mise en évidence sans utiliser les produits remarquables.**

19) $z^2 - 9 = (z - 3)(z + 3)$

20) $25a^2 - 16 = (5a - 4)(5a + 4)$

21) $16a^2 - 25y^2 = (4a - 5y)(4a + 5y)$

22) $x^2y^2z^2 - p^2 = (x\,y\,z - p)^2$

23) $\dfrac{x^2}{4} - \dfrac{y^2}{25} = (\dfrac{x}{2} - \dfrac{y}{5})(\dfrac{x}{2} + \dfrac{y}{5})$

24) $a^3 - \dfrac{4a}{9} = a(a^2 - \dfrac{4}{9}) = a(a - \dfrac{2}{3})(a + \dfrac{2}{3})$

25) $(a - 1)^2 - 1^2 = [(a - 1) - 1][(a - 1) + 1] = (a - 1 - 1)(a - 1 + 1) = a(a - 2)$

4. Même exercice

26) $a^4 - 2a^2 + 1 = (a^2 - 1)^2 = [(a - 1)(a + 1)]^2 = (a - 1)^2 (a + 1)^2$

27) $81a^4 - 169 = (9a^2 - 13)(9a^2 + 13) = (3a - \sqrt{13})(3a + \sqrt{13})(9a^2 + 13)$

27) $(5x - \frac{3}{2})^2 - \frac{81}{4} = [(5x - \frac{3}{2}) - \frac{9}{2}][(5x - \frac{3}{2}) + \frac{9}{2}] = (5x - \frac{3}{2} - \frac{9}{2})(5x - \frac{3}{2} + \frac{9}{2}) = (5x - \frac{12}{2})(5x + \frac{6}{2})$

$= (5x - 6)(5x + 3)$

28) $25x^5y - 49xy^3 = xy(25x^4 - 49y^2) = xy(5x^2 - 7y)(5x^2 + 7y)$

29) $x^2 + x + \frac{1}{4} = (x + \frac{1}{2})^2$

30) $2a - 8a^3 + 8a^5 = 2a(1 - 4a^2 + 4a^4) = 2a(1 - 2a^2)^2$

31) $64x^5 + 16x^4 + x^3 = x^3(64x^2 + 16x + 1) = x^3(8x + 1)^2$

32) $\frac{a^2}{4} + \frac{1}{49} - \frac{a}{7} = (\frac{a}{2} - \frac{1}{7})^2$

33) $(a - b)^2 - 121 = [(a - b) - 11][(a + b) + 11] = (a - b - 11)(a + b + 11)$

34) $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$

35) $(2x + 5)^2 - (3x - 2)^2 = [(2x + 5) - (3x - 2)][(2x + 5) + (3x - 2)] = (2x + 5 - 3x + 2)(2x + 5 + 3x - 2)$

$= (-x + 7)(5x + 3)$

36) $x^4 + 1 - 2x^2 = (x^2 - 1)^2 = [(x - 1)(x + 1)]^2 = (x - 1)^2 (x + 1)^2$

37) $\frac{xy}{3} + \frac{y^2}{9} + \frac{x^2}{4} = (\frac{x}{2} + \frac{y}{3})^2$

38) $49x^2 - (x - y)^2 = [(7x - (x - y)][(7x + (x - y)] = (7x - x + y)(7x + x - y) = (6x + y)(8x - y)$

39) $-x^2 + 12x - 36 = -(x^2 - 12x + 36) = -(x-6)^2$

40) $x^2 + 5x + 9$ / Ce n'est pas un produit remarquable, on peut le factoriser par la méthode du discriminant.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4.1.9}}{2.1} = \frac{-5 \pm \sqrt{25 - 36}}{2} = \frac{-5 \pm \sqrt{-11}}{2}$$

$\sqrt{-11}$ La racine carrée d'un nombre négatif n'existe pas alors c'est impossible de trouver les racines et de factoriser.

41) $81x^2 - (x-4)^2 = [(9x - (x-4)][(9x + (x-4)] = (9x - x + 4)(9x + x - 4) = (8x + 4)(10x - 4)$
$= 4(2x + 1).\ 2(5x - 2) = 8(2x + 1)(5x - 2)$

42) $(u + t)^2 - (u - t)^2 = [(u+t) - (u-t)][(u+t) + (u-t)] = (u + t - u + t)(u + t + u - t)$
$= 2t.\ 2u = 4ut$

43) $3x^5 - 48xy^8 = 3x(x^4 - 16y^8) = 3x(x^2 - 4y^4)(x^2 + 4y^4) = 3x(x - 2y^2)(x + 2y^2)(x^2 + 4y^4)$

44) $81x^4 - \dfrac{1}{16} = (9x^2 - \dfrac{1}{4})(9x^2 + \dfrac{1}{4}) = (3x - \dfrac{1}{2})(3x + \dfrac{1}{2})(9x^2 + \dfrac{1}{4})$

45) $\dfrac{1}{9} - 7x^2 = (\dfrac{1}{3} - x\sqrt{7})(\dfrac{1}{3} - x\sqrt{7})$

46) $4x^2 - (3x + 1)^2 = [(2x - (3x + 1)][(2x + (3x + 1)] = (2x - 3x - 1)(2x + 3x + 1) = (-x - 1)(5x + 1)$

47) $x^5 - 8x^3 + 16x = x(x^4 - 8x^2 + 16) = x(x^2 - 4)^2 = x[(x - 2)(x + 2)]^2 = x(x - 2)^2(x + 2)^2$

48) $16a^2 + 9b^2 - 24ab^8$ / Ce n'est pas un carré parfait et on ne sait pas mettre en évidence.

49) $\dfrac{1}{4}x^6 - \dfrac{2}{3}x^5 + \dfrac{4}{9}x^4 = x^4(\dfrac{1}{4}x^2 - \dfrac{2}{3}x + \dfrac{4}{9}) = x^4(\dfrac{1}{2}x - \dfrac{2}{3})^2$

50) $\dfrac{1}{9}a^6b^4 - \dfrac{2}{9}a^5b^2 + \dfrac{1}{9}a^4 = \dfrac{1}{9}a^4(a^2b^4 - 2ab^2 + 1) = \dfrac{1}{9}a^4(ab^2 - 1)^2$

51) $25a^2 + 10a + 1 = (5a^2 + 1)^2$

52) $x^4 + x^2y + \dfrac{y^2}{4} = (x^2 + \dfrac{y}{2})^2$

53) $x^2 - 2 + \dfrac{1}{x^2} = (x - \dfrac{1}{x})^2$

54) $100a^2 + 9b^6 - 30ab^3$ / Ce n'est pas un carré parfait, il faudrait $-60ab^3$ à la place de $-30ab^3$

55) $25x^4 + 16x^2y^4 - 40x^3y^2 = x^2(25x^2 + 16y^2 - 40xy) = x^2(5x - 4y)^2$

56) $4a^5 + 12a^3b^4 + 9ab = a(4a^4 + 12a^2b^4 + 9b)$

57) $(2a + 1)^2 - (3 - a)^2 = [(2a + 1) - (3 - a)][(2a + 1) + (3 - a)]$
$= (2a + 1 - 3 + a)(2a + 1 + 3 - a) = (3a - 2)(a + 4)$

58) $9 + \dfrac{a^2b^2}{4} - 3ab = (3 - \dfrac{ab}{2})^2$

59) $25a^2 - (b - 2a)^2 = [5a - (b - 2a)][5a + (b - 2a)] = (5a - b + 2a)(5a + b - 2a)$
$= (7a - b)(3a + b)$

5. **Factorise par la méthode du discriminant.**

$$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **Factorisation : a (x- x_1) (x − x_2)**

1) $x^2 + 3x + 2 = (x + 1)(x + 2)$

$a = 1, b = 3, c = 2$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4.1.2}}{2.1} = \dfrac{-3 \pm \sqrt{9 - 8}}{2} = \dfrac{-3 \pm \sqrt{1}}{2} = \dfrac{-3 \pm 1}{2}$ ⟹ $x_1 = \dfrac{-3+1}{2} = \dfrac{-2}{2} = -1$ et $x_2 = \dfrac{-3-1}{2} = \dfrac{-2}{2} = -2$

2) $x^2 + 8x + 15 = (x + 3)(x + 5)$

$a = 1, b = 8, c = 15$

$x = \dfrac{-8 \pm \sqrt{8^2 - 4.1.15}}{2.1} = \dfrac{-8 \pm \sqrt{64 - 60}}{2} = \dfrac{-8 \pm \sqrt{4}}{2} = \dfrac{-8 \pm 2}{2}$ ⟹ $x_1 = \dfrac{-8+2}{2} = \dfrac{-6}{2} = -3$ et $x_2 = \dfrac{-8-2}{2} = -5$

3) $x^2 + x - 6 = (x - 2)(x + 3)$

$a = 1, b = 1, c = -6$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4.1.(-6)}}{2.1} = \dfrac{-1 \pm \sqrt{1 + 24}}{2} = \dfrac{-1 \pm \sqrt{25}}{2} = \dfrac{-1 \pm 5}{2}$ ⟹ $x_1 = \dfrac{-1+5}{2} = \dfrac{4}{2} = 2$ et $x_2 = \dfrac{-1-5}{2} = -3$

4) $x^2 + 5x - 6 = $ **(x - 1) (x + 6)**

a = 1, b = 5, c = -6

$$x = \frac{-5 \pm \sqrt{5^2 - 4.1.(-6)}}{2.1} = \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2} = \frac{-5 \pm 7}{2} \implies x_1 = \frac{-5+7}{2} = \frac{2}{2} = 1$$

$$x_2 = \frac{-5-7}{2} = -6$$

5) $x^2 + 4x - 21 = $ **(x - 3) (x + 7)**

a = 1, b = 4, c = -21

$$x = \frac{-4 \pm \sqrt{4^2 - 4.1.(-21)}}{2.1} = \frac{-4 \pm \sqrt{16 + 84}}{2} = \frac{-4 \pm \sqrt{100}}{2} = \frac{-4 \pm 10}{2} \implies x_1 = \frac{-4+10}{2} = 3$$

$$x_2 = \frac{-4-10}{2} = -7$$

6) $x - 8x + 12 = $ **(x - 6) (x - 2)**

a = 1, b = -8, c = 12

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4.1.12}}{2.1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2} \implies x_1 = \frac{8+4}{2} = 6 \text{ et } x_2 = \frac{8-4}{2} = 2$$

7) $x^2 + 13x - 14 = $ **(x - 14) (x + 1)**

a = 1, b = 13, c = -14

$$x = \frac{13 \pm \sqrt{13^2 - 4.1.(-14)}}{2.1} = \frac{13 \pm \sqrt{169 + 56}}{2} = \frac{13 \pm \sqrt{225}}{2} = \frac{13 \pm 15}{2} \implies x_1 = \frac{13+15}{2} = 14$$

$$x_2 = \frac{13-15}{2} = -1$$

8) $2x^2 - 14x + 24 = $ **2(x - 4) (x - 3)**

a = 2, b = -14, c = 24

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4.2.24}}{2.2} = \frac{14 \pm \sqrt{196 - 192}}{4} = \frac{14 \pm \sqrt{4}}{4} = \frac{14 \pm 2}{4} \implies x_1 = \frac{14+2}{4} = 4$$

$$x_2 = \frac{14-2}{4} = 3$$

9) $3x^2 + 3x - 6 = $ **3(x - 1) (x + 2)**

a = 3, b = 3, c = -6

$$x = \frac{-3 \pm \sqrt{3^2 - 4.3.(-6)}}{2.3} = \frac{-3 \pm \sqrt{9 + 72}}{6} = \frac{-3 \pm \sqrt{81}}{6} = \frac{-3 \pm 9}{6} \implies x_1 = \frac{-3+9}{6} = 1 \text{ et } x_2 = \frac{-3-9}{6} = -2$$

10) $3x^2 - 3x - 6 = $ **(x - 6) (x + 1)**

a = 3, b = -3, c = -6

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4.3.(-6)}}{2.3} = \frac{3 \pm \sqrt{9 + 72}}{6} = \frac{3 \pm \sqrt{81}}{6} = \frac{3 \pm 9}{6} \implies x_1 = \frac{3+9}{2} = 6 \text{ et } x_2 = \frac{3-9}{6} = -1$$

6. Exercices récapitulatifs : factorise (avec la méthode de ton choix)

1) $9x^2 + 2x + \dfrac{1}{9} = (3x + \dfrac{1}{3})^2$

2) $2x^6 + 2 - 4x^3 = 2(x^6 - 2x^3 + 1) = 2(x^3 - 1)^2$

3) $x^4 - 2x^3 - x + 2 = (x^4 - x) - (2x^3 - 2) = x(x^3 - 1) - 2(x^3 - 1) = (x^3 - 1)(x - 2)$

4) $3(x - y) - x(y - x) = 3(x - y) + x(x - y) = (x - y)(3 + x)$

5) $y^5 - \dfrac{2y^3}{3} + \dfrac{y}{9} = y(y^4 - \dfrac{2y^2}{3} + \dfrac{1}{9}) = y(y^2 - \dfrac{1}{3})^2$

6) $81x^4 - 144 = 9(9x^4 - 16) = 9(3x^2 - 4)(3x^2 + 4) =$

7) $(2x + 1)^2 - (x + 1)^2 = [(2x + 1) - (x + 1)][(2x + 1) + (x + 1)]$

   $= (2x + 1 - x - 1)(2x + 1 + x + 1) = x(3x + 2)$

8) $5t^4 - 10t^2 + 5 = 5(t^4 - 2t^2 + 1) = 5(t^2 - 1)^2 = 5[(t - 1)(t + 1)]^2 = 5(t - 1)^2(t + 1)^2$

9) $121a^2 - (2a + 3)^2 = [11a - (2a + 3)][11a + (2a + 3)] = (11a - 2a - 3)(11a + 2a + 3)$

   $= (9a - 3)(13a + 3) = 3(3a - 1)(13a + 3)$

10) $x^2 + 4y^2 - 1 - 4xy = (x^2 - 4xy + 4y^2) - 1 = (x - 2y)^2 - 1 = [(x - 2y) - 1][(x - 2y) + 1]$

    $= (x - 2y - 1)(x - 2y + 1)$

11) $u^3 - 6u^2 - u + 6 = (u^3 - 6u^2) + (-u + 6) = u^2(u - 6) - (u - 6) = (u - 6)(u^2 - 1)$

    $= (u - 6)(u - 1)(u + 1)$

12) $-u^2 + 2u - 1 = -(u^2 - 2u + 1) = -(u - 1)^2$

13) $8a^3 - 8a^2 + 2a = 2a(4a^2 - 4a + 1) = 2a(2a - 1)^2$

14) $a^3 - 3a^2 - a + 3 = (a^3 - 3a^2) + (-a + 3) = a^2(a - 3) - (a - 3) = (a - 3)(a^2 - 1)$

    $= (a - 3)(a - 1)(a + 1)$

15) $x^2 - 12x - y^2 + 36 = (x^2 - 12x + 36) - y^2 = (x - 6)^2 - y^2 = (x - 6 - y)(x - 6 + y)$

16) $xy^2 - xz^2 = $ **x (y² - z²) = x (y - z) (y + z)**

17) $x^4 y^2 - x^2y^4 = $ **y² x² (x² - y²) = y² x² (x - y) (x + y)**

18) $x^2y - 6xy + 9y = $ **y (x² - 6x + 9) = y (x - 3) ²**

19) $3x^3y - 3xy^3 = $ **3xy (x² - y²) = 3xy (x - y) (x + y)**

20) $0,04y^2 - 0,09x^2 = $ **(0,2x - 0,3y) (0,2x - 0,3y)**

21) $x^8 - y^8 = $ **(x⁴ - y⁴) (x⁴ + y⁴) = (x² - y²) (x² + y²) (x⁴ + y⁴) = (x - y) (x + y) (x² + y²) (x⁴ + y⁴)**

22) $x^2 - x + 4^{-1} = $ **x² - x + ¼ = (x - ½) ²   ou   = x² - x + 0,25 = (x - 0,5) ²**

23) $mn^4op^4 - mn^2op^2 = $ **mn²op² (n²p² - 1) = mn²op² (n p - 1) (n p + 1)**

24) $2a^2 - 12a + 18 = $ **2(a² - 6a + 9) = 2(a - 3) ²**

25) $x^6 - \dfrac{1}{16} = $ **(x³ - ¼) (x³ - ¼) = (x³ - 0,25) (x³ - 0,25)**

26) $a^4 - 2a^2 + 1 = $ **(a² - 1) ² = [(a - 1) (a + 1)] ² = (a - 1) ² (a + 1) ²**

27) $5x^3 - 15xy^2 = $ **5x (x² - 3y²)**

28) $a^4 - 6a^2 + 9 = $ **(a² - 3) ²**

29) $49x^3 - x = $ **x (49x² -1) = x (7x - 1) (7x - 1)**

30) $\dfrac{x^2}{4} + \dfrac{1}{49} - \dfrac{x}{7} = $ **($\dfrac{x}{2}$) ² - $\dfrac{x}{7}$ + ($\dfrac{1}{7}$) ² = ($\dfrac{x}{2} - \dfrac{1}{7}$) ²**

31) $9x^2y^2 - 16 = $ **(3xy - 4) (3xy + 4)**

32) $4x^2 - 2x + 1 = $ **(2x - 1) ²**

33) $4a^2b^3 - b = $ **b (4a²b² - 1) = b (2ab - 1) (2ab +1)**

34) $75a^2 + 30a + 3 = 3(25a^2 + 10a + 1) = 3(5a + 1)^2$

35) $(x + \sqrt{3})^2 - 16 = (x + \sqrt{3} - 4)(x + \sqrt{3} + 4)$

36) $4x^2(x + 3) - (3 + x) = (x + 3)(4x^2 - 1) = (x + 3)(2x - 1)(2x + 1)$

37) $3x^3y^2 - 5x^5 = x^3(3y^2 - 5x^2)$

38) $9(x^2 - 3) - y^2(x^2 - 3) = (x^2 - 3)(9 - y^2) = (x^2 - 3)(3 - y)(3 + y)$

39) $3x(y^2 - 2y) + 3x = 3x[(y^2 - 2y) + 1] = 3x(y^2 - 2y + 1) = 3x(y - 1)^2$

40) $x^4 - 2x^2 + 1 = (x^2 - 1)^2 = [(x - 1)(x + 1)]^2 = (x - 1)^2(x + 1)^2$

41) $(4x^2 + 12x)^2 + 9(4x^2 + 12x) = (4x^2 + 12x)^2[(4x^2 + 12x) + 9] = (4x^2 + 12x)^2(4x^2 + 12x + 9)$

$= [4x(x + 3)]^2(2x + 3)^2 = (4x)^2(x + 3)^2(2x + 3)^2$

$= 16x^2(x + 3)^2(2x + 3)^2$

42) $3x^4(x^2 - \dfrac{4}{3}) - 75(x^2 - \dfrac{4}{3}) = 3(x^2 - \dfrac{4}{3})(x^4 - 25) = 3(x^2 - \dfrac{4}{3})(x^2 - 5)(x^2 + 5)$

7. **Exercices supplémentaires (plus difficiles)**

1) $2a(x + y) - 3b(-x - y) = 2a(x + y) + 3b(x + y) = (x + y)(2a + 3b)$

2) $x(2a - b) + y(b - 2a) = x(2a - b) - y(2a - b) = (2a - b)(x - y)$

3) $a(x - y) - (y - x) = a(x - y) + (x - y) = (x - y)(a + 1)$

4) $(4a - 2b)(2x - 3y) + (3y - 2x)(b - 2a) = 2(2a - b)(2x - 3y) + (2x - 3y)(2a - b)$

$= 3(2a - b)(2x - 3y)$

5) $a^2(x - 1)(a + b) + a^3(1 - x) = a^2[(x - 1)(a + b) - a(x - 1)] = a^2(x - 1)[(a + b) - a]$

$= a^2(x - 1)b = a^2b(x - 1)$

6) $(x – 2y) (a – b) – (b – a) (2x + y) = (x – 2y) (a – b) + (a - b) (2x + y)$

$$= (a – b) (x – 2y + 2x + y) = (a – b) (3x – y)$$

7) $5x^2 - 5 = 5(x^2 – 1) = 5(x – 1) (x + 1)$

8) $a^4x^4 – a^4 = a^4 (x^4 – 1) = a^4 (x^2 – 1) (x^2 + 1) = a^4 (x – 1) (x + 1) (x^2 + 1)$

9) $a^4b^2x^5 – a^2x = a^2x (a^2b^2x^4 – 1) = a^2x (abx^2 – 1) (abx^2 + 1)$

10) $x^5y^7 – x y = x y (x^4 y^6 – 1) = x y (x^2y^3 – 1) (x^2y^3 + 1)$

11) $(x + 2)^3 - 3(x + 2) = (x + 2) [(x + 2)^2 – 3] = (x + 2) (x^2 + 4x + 4 – 3) = (x + 2) (x^2 + 4x + 1)$

12) $(a – b) – (a – b) x^4 = (a – b) (1 – x^4) = (a – b) (1 – x^2) (1 + x^2)$

$$= (a – b) (1 – x) (1 + x) (1 + x^2)$$

13) $–3x^9 + 3x^5 = 3x^5 (-1 + x^2) = 3x^5 (x^2 – 1) = 3x^5 (x – 1) (x + 1)$

14) $2x^3 - 7x = x (2x^2 – 7)$

15) $16a^4b^2 - 24a^2b^3 + 9b^4 = b^2 (16a^4 - 24a^2b + 9b^2) = b^2 (4a^2 – 3b)^2$

16) $x^2(a^2 - 4) – (a^2 - 4) = (a^2 - 4) (x^2 – 1) = (a - 2) (a + 2) (x – 1) (x + 1)$

17) $(a + 1)^4 – (a + 1)^2 = (a + 1)^2 [(a + 1)^2 - 1] = (a + 1)^2 [(a + 1) - 1] [(a + 1) + 1]$

$$= (a + 1)^2 a (a + 2) = a (a + 1)^2 (a + 2)$$

18) $125x^3(x– y)^2 - 45x (3x + 2y)^2 = 5x [25(x– y)^2 - 9(3x + 2y)^2] = 5x [5^2(x– y)^2 - 3^2(3x + 2y)^2]$

$$= 5x\left[[5(x– y)]^2 - [3(3x + 2y)]^2 \right]$$

$$= 5x [5(x – y) – 3(3x + 2y)] [5(x – y) + 3(3x + 2y)]$$

$$= 5x (5x – 5y – 9x - 6y) (5x – 5y + 9x + 6y)$$

$$= 5x (-4x – 11y) (14x – y) = -5x (4x + 11y) (14x – y)$$

19) $(a^2 + b^2 - c^2)^2 - (a^2 - b^2 + c^2)^2 = [(a^2 + b^2 - c^2) - (a^2 - b^2 + c^2)] [(a^2 + b^2 - c^2) + (a^2 - b^2 + c^2)]$

$$= (a^2 + b^2 - c^2 - a^2 + b^2 - c^2) (a^2 + b^2 - c^2 + a^2 - b^2 + c^2)$$

$$= (2b^2 - 2c^2) 2a^2 = 2(b^2 - c^2). 2a^2 = 4a^2 (b^2 - c^2) = 4a^2 (b - c) (b + c)$$

20) $\dfrac{(a-1)^2}{3} - \dfrac{a^2}{12} = \dfrac{1}{3}\left[(a-1)^2 - \dfrac{a^2}{4}\right] = \dfrac{1}{3}\left[(a-1)^2 - \left(\dfrac{a}{2}\right)^2\right] = \dfrac{1}{3}\left(a-1-\dfrac{a}{2}\right)\left(a-1+\dfrac{a}{2}\right)$

$$= \dfrac{1}{3}\left(\dfrac{2a}{2}-1-\dfrac{a}{2}\right)\left(\dfrac{2a}{2}-1+\dfrac{a}{2}\right) = \dfrac{1}{3}\left(\dfrac{a}{2}-1\right)\left(\dfrac{3a}{2}-1\right)$$

21) $x^2 - 8x + 12 = (x-6)(x-2)$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\qquad$ $a=1, b=-8, c=12$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4.1.12}}{2.1} = \dfrac{8 \pm \sqrt{64-48}}{2} = \dfrac{8 \pm \sqrt{16}}{2} = \dfrac{8 \pm 4}{2}$ $\Rightarrow$ $x_1 = \dfrac{8+4}{2} = \dfrac{12}{2} = 6$ et $x_2 = \dfrac{8-4}{2} = \dfrac{4}{2} = 2$

22) $x^2 - 14x + 13 = (x-13)(x-1)$

$a=1, b=-14, c=13$

$x = \dfrac{14 \pm \sqrt{(-14)^2 - 4.1.13}}{2.1} = \dfrac{14 \pm \sqrt{196-52}}{2} = \dfrac{14 \pm \sqrt{144}}{2} = \dfrac{14 \pm 12}{2}$ $\Rightarrow$ $x_1 = \dfrac{14+12}{2} = 13$

$x_2 = \dfrac{14-12}{2} = 1$

23) $x^2 - 22x + 85 = (x-17)(x-5)$

$a=1, b=-22, c=85$

$x = \dfrac{22 \pm \sqrt{(-22)^2 - 4.1.85}}{2.1} = \dfrac{22 \pm \sqrt{484-340}}{2} = \dfrac{22 \pm \sqrt{144}}{2} = \dfrac{22 \pm 12}{2}$ $\Rightarrow$ $x_1 = \dfrac{22+12}{2} = \dfrac{34}{2} = 17$

$x_2 = \dfrac{22-12}{2} = \dfrac{10}{2} = 5$

24) $x^2 - 4x - 5 = (x-5)(x+1)$

$a=1, b=-4, c=-5$

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4.1.(-5)}}{2.1} = \dfrac{4 \pm \sqrt{16+20}}{2} = \dfrac{4 \pm \sqrt{36}}{2} = \dfrac{4 \pm 6}{2}$ $\Rightarrow$ $x_1 = \dfrac{4+6}{2} = \dfrac{10}{2} = 5$ et $x_2 = \dfrac{4-6}{2} = \dfrac{-2}{2} = -1$

25) $x^2 + 10x + 16 = (x+2)(x+8)$

$x = \dfrac{-10 \pm \sqrt{10^2 - 4.1.16}}{2.1} = \dfrac{-10 \pm \sqrt{100-64}}{2} = \dfrac{-10 \pm \sqrt{36}}{2} = \dfrac{-10 \pm 6}{2}$ $\Rightarrow$ $x_1 = \dfrac{-10+6}{2} = -2$

$x_2 = \dfrac{-10-6}{2} = -8$

26) $x^2 - 115x + 1500 = (x-100)(x-15)$

$x = \dfrac{115 \pm \sqrt{(-115)^2 - 4.1.1500}}{2.1} = \dfrac{115 \pm \sqrt{13225-6000}}{2} = \dfrac{115 \pm \sqrt{7225}}{2} = \dfrac{115 \pm 85}{2}$ $\Rightarrow$ $x_1 = \dfrac{200}{2} = 100$

$x_2 = \dfrac{30}{2} = 15$

27) $x^2 - 4x - 32 = (x-8)(x+4)$

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4.1.(-32)}}{2.1} = \dfrac{4 \pm \sqrt{16+128}}{2} = \dfrac{4 \pm \sqrt{144}}{2} = \dfrac{4 \pm 12}{2}$ $\Rightarrow$ $x_1 = \dfrac{16}{2} = 8$ et $x_2 = \dfrac{-8}{2} = -4$

28) $x^2 + 5x - 14 = (x - 2)(x + 7)$

$$x = \frac{-5 \pm \sqrt{5^2 - 4.1.(-14)}}{2.1} = \frac{-5 \pm \sqrt{25 + 56}}{2} = \frac{-5 \pm \sqrt{81}}{2} = \frac{-5 \pm 9}{2} \implies x_1 = \frac{4}{2} = 2 \text{ et } x_2 = \frac{-14}{2} = -7$$

29) $x^2 + 20x + 19 = (x - 19)(x + 1)$

$$x = \frac{20 \pm \sqrt{20^2 - 4.1.19}}{2.1} = \frac{20 \pm \sqrt{400 - 76}}{2} = \frac{20 \pm \sqrt{324}}{2} = \frac{20 \pm 18}{2} \implies x_1 = \frac{38}{2} = 19 \text{ et } x_2 = \frac{-2}{2} = -1$$

30) $x^2 - 4x - 12 = (x - 6)(x + 2)$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4.1.(-12)}}{2.1} = \frac{4 \pm \sqrt{16 + 48}}{2} = \frac{4 \pm \sqrt{64}}{2} = \frac{4 \pm 8}{2} \implies x_1 = \frac{12}{2} = 6 \text{ et } x_2 = \frac{-4}{2} = -2$$

8. **Autres exercices**

1) $\dfrac{2^{12}}{12^2} = \dfrac{2^{12}}{(2.\,2.\,3)^2} = \dfrac{2^{12}}{(2^2.\,3)^2} = \dfrac{2^{12-4}}{2^4.\,3^2} = \dfrac{2^8}{3^2}$ a) $\dfrac{2^8}{3^2}$ b) $\dfrac{2^{10}}{2}$ c) $\dfrac{2^6}{6}$ d) $\dfrac{1}{6}$ e) 1

2) **Si on écrit $3^5.\,27^2.\,3^2.\,243$ sous la forme d'une puissance de 3, l'exposant sera égal à :**

   a) 10   b) 18   c) 21   d) 30   e) 450

   $3^5.\,27^2.\,3^2.\,243 = 3^{5+2}.\,(3.\,9)^2.\,243 = 3^7.\,(3.\,3.\,3)^2.\,(81.\,3) = 3^7.\,(3^3)^2.\,(9^2.3)$
   $= 3^7.\,3^{3.2}.\,((3^2)^2.\,3) = 3^7.\,3^6.\,3^4.\,3 = 3^{7+6+4+1} = 3^{18}$

3) $6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6 =$ a) 6^0 b) 6^7 c) 36^6 d) 6^6 e) 36^{36}

   $= 6.\,6^6 = 6^{1+6} = 6^7$

4) $\dfrac{2^{13} + 2^{14} + 2^{15}}{14} =$ a) 2^{28} b) 2^{14} c) 2^{12} d) 2^{10} e) 2^6

   $= \dfrac{2^{13}.\,(1 + 2 + 2^2)}{2.7} = \dfrac{2^{12}.\,(1 + 2 + 4)}{7} = \dfrac{2^{12}.\,7}{7} = 2^{12}$