

Correctif factorisation



Synthèse

Binômes

- 1) mise en évidence
- 2) produit remarquable : $a^2 - b^2 = (a + b)(a - b)$

Trinômes

- 1) mise en évidence
- 2) produits remarquables : $a^2 + 2ab + b^2 = (a + b)^2$
 $a^2 - 2ab + b^2 = (a - b)^2 = (b - a)^2$

- 3) discriminant : $ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadri nômes (et +)

- 1) mise en évidence
- 2) produits remarquables
- 3) discriminant

Remarques : il faut toujours factoriser au maximum !
 $a^2 + b^2$: pas moyen de factoriser



Exercices

1. Factorise à l'aide de la mise en évidence

$$1) 15a^7b^2 - 10a^5b^3 = 5a^5b^2 (3a^2 - 2b)$$

$$2) 12x^2y^2 - 18xy^3 + 24x^3y = 6xy (2y - 3y^2 + 4x^2)$$

$$3) (x + 1)^2 - 3(x + 1) = (x + 1) (x + 1 - 3) = (x + 1) (x - 2)$$

$$4) 2(n + 1)n + 3(n + 1) = (n + 1) (2n + 3)$$

$$5) a(x - y) - (y - x)^2 = a(x - y) - (x - y)^2 = (x - y) [(a - (x - y))] = (x - y) (a - x + y)$$

$$6) (x + 2)(x - 3) - (x - 3)(x + 1) = (x - 3) [(x + 2) - (x + 1)] \\ = (x - 3) (x + 2 - x - 1) = (x - 3) 1 = (x - 3)$$

$$7) 3(x - 2)x - (x + 2)^2 + (x + 2) = 3x(x - 2) + (x + 2) [-(x + 2) + 1] \\ = 3x(x - 2) + (x + 2) (-x - 2 + 1) \\ = 3x(x - 2) + (x + 2) (-x - 1)$$

$$8) 35x^3y^2 - 49x^2y^3 = 7x^2y^2 (5x - 7y)$$

$$9) 3x(x + y)^3 - 4x^2(x + y)^2 = x(x + y)^2 (3(x + y) - 4x) = x(x + y)^2 (3x + 3y - 4x) \\ = x(x + y)^2 (-x + 3y)$$

$$10) y(b - a) + x(a - b) = y(b - a) - x(-a + b) = y(b - a) - x(b - a) = (y - x)(b - a)$$

$$11) 5x(x + 1)^2 - 10x^2(x + 1) = (x + 1) (5x(x + 1) - 10x^2) = (x + 1) (5x^2 + 5x - 10x^2) \\ = (x + 1) (-5x^2 + 5x) = 5x(x + 1) (-x + 1)$$

$$12) 6x^2(u - 1) - 4x(1 - u) = 2x(u - 1) (3x - 2)$$

$$13) x(2a - b) + (b - 2a) = x(2a - b) - (-b + 2a) = x(2a - b) - (2a - b) = (2a - b)(x - 1)$$

2. Même exercice

1) $ab - b^2 = b(a - b)$

2) $xy + y = y(x + 1)$

3) $a^2b - ab^2 = ab(a - b)$

4) $xy - 2y = y(x - 2)$

5) $x^3y^2 - x^2y^3 = x^2y^2(x - y)$

6) $6x^2y + 4xy^2 = 2xy(3x + 2y)$

7) $15x^7b^2 - 10x^5b^3 = 5x^5b^2(3x^2 - 2b)$

8) $Y(b - a) + b(b - a) = (b - a)(y + b)$

9) $3(x + y) - 2(x - y) = 3x + 3y - 2x + 2y = x + 5y$

10) $12a^2b^2 - 18ab^3 + 24a^3b = 6ab(2ab - 3b^2 + 4a^2)$

11) $12x^2y^3 - 30x^3y^2 + 18xy^4 = 6xy^2(2xy - 5x^2 + 3y^2)$

12) $3x^2 - 9xy^2 + 6x = 3x(x - 3y^2 + 6)$

13) $3xyz^3 - 21x^2y^2z^2 - 6x^3y^3z = 3xyz(z^2 - 7xyz - 2x^2y^2)$

14) $5a(a + 2)^2 - 3a^2(a + 2) = a(a + 2)[5(a + 2) - 3a] = a(a + 2)(5a + 10 - 3a)$
 $= a(a + 2)(2a + 10) = 2a(a + 2)(a + 5)$

15) $5(x + y - z) - 10x(x + y - z) = (x + y - z)(5 - 10x) = 5(x + y - z)(1 - 2x)$

16) $45x^3y^4z^5 + 60x^5y^2z - 90x^4y^3z^2 = 15x^3y^2z(3y^2z^4 + 4x^2 - 6xyz)$

17) $39a^5b^5c^3 - 65a^5b^3cd = a^5b^3c(39b^2c^2 - 65d)$

$$18) -51a^4b^5c + 17a^3b^2c - 32a^3b^5c^4 = a^3b^2c (-51ab^3 + 17 - 32b^3c^3)$$

$$19) a(x - y) + b(x - y) = (x - y) (a + b)$$

$$20) 2a (b - c) - 4c (b - c) = 2(b - c) (a - 2c)$$

$$21) 9a^2b(x + y) + 3ab^2(x + y) = 3ab(x + y) (3a + b)$$

$$22) a (m - n) - b (n - m) = a (m - n) + b (m - n) = (m - n) (a + b)$$

$$23) 5a^2 (b - 2) + 15a (2 - b) = 5a (a (b - 2) - 3(b - 2)) = 5a (b - 2) (a - 3)$$

3. Factorise à l'aide des produits remarquables (théorie : p.1 et 2)

$$1) 1/9 - x^2 = (1/3 - x) (1/3 + x)$$

$$2) x^2 - 144 = (x - 12) (x + 12)$$

$$3) a^2 - 4a - 4 = (a + 4,87) (a - 0,85)$$

Ce n'est pas un produit remarquable.

On peut utiliser la méthode du discriminant $A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$A = \frac{-4 \pm \sqrt{(-4)^2 - 4.1.(-4)}}{2.1} = \frac{-4 \pm \sqrt{(16+16)}}{2} = \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 5,7}{2} = \frac{-4 \pm 5,7}{2} \Rightarrow A_1 = \frac{-4 + 5,7}{2} = \frac{1,7}{2} = 0,85$$

$$A_2 = \frac{-4 - 5,7}{2} = \frac{-9,7}{2} = -4,87$$

$$4) x^2 - 8x + 16 = (x - 4)^2$$

$$5) 25x^2 + 30x + 9 = (5x + 3)^2$$

$$6) x^2 - \frac{2x}{6} + \frac{1}{9} = (x - \frac{1}{3})^2$$

$$7) 49 - 14a + a^2 = (7 - a)^2$$

$$8) 9a^2 - 12ab + 4b^2 = (3a - 2b)^2$$

$$9) x^2 + 4xy + 4y^2 = (x + 2y)^2$$

$$10) \frac{a^2}{9} + \frac{2ab}{15} + \frac{b^2}{25} = \left(\frac{a}{3} + \frac{b}{5}\right)^2$$

$$11) 16x^2 - 4 = 4(4x^2 - 1) = 4(x - 1)(x + 1)$$

$$12) a^4 - 81 = (a^2 - 9)(a^2 + 9) = (a - 3)(a + 3)(a^2 + 9)$$

$$13) 1 + 2x^2 + x^4 = (1 + x^2)^2$$

$$14) x^2 - 4xy + 4y^2 = (x - 2y)^2$$

$$15) x^6 - 6x^3 + 9 = (x^3 - 3)^2$$

$$16) x^4 - 18x^2 + 81 = (x^2 - 9)^2 = [(x - 3)(x + 3)]^2 = (x - 3)^2(x + 3)^2$$

$$17) 4x^2 - 28x + 49 = (2x - 7)^2$$

$$18) xy^2 - y^2z = y^2(x - z) \text{ C'est par la mise en évidence sans utiliser les produits remarquables.}$$

$$19) z^2 - 9 = (z - 3)(z + 3)$$

$$20) 25a^2 - 16 = (5a - 4)(5a + 4)$$

$$21) 16a^2 - 25y^2 = (4a - 5y)(4a + 5y)$$

$$22) x^2y^2z^2 - p^2 = (xyz - p)^2$$

$$23) \frac{x^2}{4} - \frac{y^2}{25} = \left(\frac{x}{2} - \frac{y}{5}\right)\left(\frac{x}{2} + \frac{y}{5}\right)$$

$$24) a^3 - \frac{4a}{9} = a\left(a^2 - \frac{4}{9}\right) = a\left(a - \frac{2}{3}\right)\left(a + \frac{2}{3}\right)$$

$$25) (a - 1)^2 - 1^2 = [(a - 1) - 1][(a - 1) + 1] = (a - 1 - 1)(a - 1 + 1) = a(a - 2)$$

4. Même exercice

$$26) a^4 - 2a^2 + 1 = (a^2 - 1)^2 = [(a - 1)(a + 1)]^2 = (a - 1)^2 (a + 1)^2$$

$$27) 81a^4 - 169 = (9a^2 - 13)(9a^2 + 13) = (3a - \sqrt{13})(3a + \sqrt{13})(9a^2 + 13)$$

$$27) (5x - \frac{3}{2})^2 - \frac{81}{4} = [(5x - \frac{3}{2}) - \frac{9}{2}] [(5x - \frac{3}{2}) + \frac{9}{2}] = (5x - \frac{3}{2} - \frac{9}{2})(5x - \frac{3}{2} + \frac{9}{2}) = (5x - \frac{12}{2})(5x + \frac{6}{2}) \\ = (5x - 6)(5x + 3)$$

$$28) 25x^5y - 49xy^3 = xy(25x^4 - 49y^2) = xy(5x^2 - 7y)(5x^2 + 7y)$$

$$29) x^2 + x + \frac{1}{4} = (x + \frac{1}{2})^2$$

$$30) 2a - 8a^3 + 8a^5 = 2a(1 - 4a^2 + 4a^4) = 2a(1 - 2a^2)^2$$

$$31) 64x^5 + 16x^4 + x^3 = x^3(64x^2 + 16x + 1) = x^3(8x + 1)^2$$

$$32) \frac{a^2}{4} + \frac{1}{49} - \frac{a}{7} = (\frac{a}{2} - \frac{1}{7})^2$$

$$33) (a - b)^2 - 121 = [(a - b) - 11][(a + b) + 11] = (a - b - 11)(a + b + 11)$$

$$34) a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$$

$$35) (2x + 5)^2 - (3x - 2)^2 = [(2x + 5) - (3x - 2)][(2x + 5) + (3x - 2)] = (2x + 5 - 3x + 2)(2x + 5 + 3x - 2) \\ = (-x + 7)(5x + 3)$$

$$36) x^4 + 1 - 2x^2 = (x^2 - 1)^2 = [(x - 1)(x + 1)]^2 = (x - 1)^2 (x + 1)^2$$

$$37) \frac{xy}{3} + \frac{y^2}{9} + \frac{x^2}{4} = (\frac{x}{2} + \frac{y}{3})^2$$

$$38) 49x^2 - (x - y)^2 = [(7x - (x - y))][(7x + (x - y))] = (7x - x + y)(7x + x - y) = (6x + y)(8x - y)$$

$$39) -x^2 + 12x - 36 = -(x^2 - 12x + 36) = -(x - 6)^2$$

40) $x^2 + 5x + 9$ Ce n'est pas un produit remarquable, on peut le factoriser par la méthode du discriminant.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} = \frac{-5 \pm \sqrt{25 - 36}}{2} = \frac{-5 \pm \sqrt{-11}}{2}$$

$\sqrt{-11}$ La racine carrée d'un nombre négatif n'existe pas alors c'est impossible de trouver les racines et de factoriser.

$$41) 81x^2 - (x - 4)^2 = [(9x - (x - 4))][(9x + (x - 4))] = (9x - x + 4)(9x + x - 4) = (8x + 4)(10x - 4) \\ = 4(2x + 1) \cdot 2(5x - 2) = 8(2x + 1)(5x - 2)$$

$$42) (u + t)^2 - (u - t)^2 = [(u + t) - (u - t)][(u + t) + (u - t)] = (u + t - u + t)(u + t + u - t) \\ = 2t \cdot 2u = 4ut$$

$$43) 3x^5 - 48xy^8 = 3x(x^4 - 16y^8) = 3x(x^2 - 4y^4)(x^2 + 4y^4) = 3x(x - 2y^2)(x + 2y^2)(x^2 + 4y^4)$$

$$44) 81x^4 - \frac{1}{16} = (9x^2 - \frac{1}{4})(9x^2 + \frac{1}{4}) = (3x - \frac{1}{2})(3x + \frac{1}{2})(9x^2 + \frac{1}{4})$$

$$45) \frac{1}{9} - 7x^2 = (\frac{1}{3} - x\sqrt{7})(\frac{1}{3} + x\sqrt{7})$$

$$46) 4x^2 - (3x + 1)^2 = [(2x - (3x + 1))][(2x + (3x + 1))] = (2x - 3x - 1)(2x + 3x + 1) = (-x - 1)(5x + 1)$$

$$47) x^5 - 8x^3 + 16x = x(x^4 - 8x^2 + 16) = x(x^2 - 4)^2 = x[(x - 2)(x + 2)]^2 = x(x - 2)^2(x + 2)^2$$

48) $16a^2 + 9b^2 - 24ab^8$ Ce n'est pas un carré parfait et on ne sait pas mettre en évidence.

$$49) \frac{1}{4}x^6 - \frac{2}{3}x^5 + \frac{4}{9}x^4 = x^4(\frac{1}{4}x^2 - \frac{2}{3}x + \frac{4}{9}) = x^4(\frac{1}{2}x - \frac{2}{3})^2$$

$$50) \frac{1}{9}a^6b^4 - \frac{2}{9}a^5b^2 + \frac{1}{9}a^4 = \frac{1}{9}a^4(a^2b^4 - 2ab^2 + 1) = \frac{1}{9}a^4(ab^2 - 1)^2$$

$$51) 25a^2 + 10a + 1 = (5a + 1)^2$$

$$52) x^4 + x^2y + \frac{y^2}{4} = (x^2 + \frac{y}{2})^2$$

$$53) x^2 - 2 + \frac{1}{x^2} = (x - \frac{1}{x})^2$$

54) $100a^2 + 9b^6 - 30ab^3$ / Ce n'est pas un carré parfait, il faudrait avoir
 $-60ab^3$ à la place de $-30ab^3$

55) $25x^4 + 16x^2y^4 - 40x^3y^2 = x^2(25x^2 + 16y^2 - 40xy) = x^2(5x - 4y)^2$

56) $4a^5 + 12a^3b^4 + 9ab = a(4a^4 + 12a^2b^4 + 9b)$

57) $(2a + 1)^2 - (3 - a)^2 = [(2a + 1) - (3 - a)][(2a + 1) + (3 - a)]$
 $= (2a + 1 - 3 + a)(2a + 1 + 3 - a) = (3a - 2)(a + 4)$

58) $9 + \frac{a^2b^2}{4} - 3ab = (3 - \frac{ab}{2})^2$

59) $25a^2 - (b - 2a)^2 = [5a - (b - 2a)][5a + (b - 2a)] = (5a - b + 2a)(5a + b - 2a)$
 $= (7a - b)(3a + b)$

5. Factorise par la méthode du discriminant.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Factorisation : } a(x - x_1)(x - x_2)$$

1) $x^2 + 3x + 2 = (x + 1)(x + 2)$

$a = 1, b = 3, c = 2$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{-3 \pm \sqrt{9 - 8}}{2} = \frac{-3 \pm \sqrt{1}}{2} = \frac{-3 \pm 1}{2} \Rightarrow x_1 = \frac{-3+1}{2} = \frac{-2}{2} = -1 \text{ et } x_2 = \frac{-3-1}{2} = \frac{-4}{2} = -2$$

2) $x^2 + 8x + 15 = (x + 3)(x + 5)$

$a = 1, b = 8, c = 15$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1} = \frac{-8 \pm \sqrt{64 - 60}}{2} = \frac{-8 \pm \sqrt{4}}{2} = \frac{-8 \pm 2}{2} \Rightarrow x_1 = \frac{-8+2}{2} = \frac{-6}{2} = -3 \text{ et } x_2 = \frac{-8-2}{2} = \frac{-10}{2} = -5$$

3) $x^2 + x - 6 = (x - 4)(x + 3)$

$a = 1, b = 1, c = -6$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1 + 24}}{2} = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} \Rightarrow x_1 = \frac{-1+5}{2} = \frac{4}{2} = 2 \text{ et } x_2 = \frac{-1-5}{2} = \frac{-6}{2} = -3$$

4) $x^2 + 5x - 6 = (x - 1)(x + 6)$

$a = 1, b = 5, c = -6$

$$x = \frac{-5 \pm \sqrt{5^2 - 4.1.(-6)}}{2.1} = \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2} = \frac{-5 \pm 7}{2} \Rightarrow x_1 = \frac{-5+7}{2} = \frac{2}{2} = 1$$

$$x_2 = \frac{-5-7}{2} = -6$$

5) $x^2 + 4x - 21 = (x - 3)(x + 7)$

$a = 1, b = 4, c = -21$

$$x = \frac{-4 \pm \sqrt{4^2 - 4.1.(-21)}}{2.1} = \frac{-4 \pm \sqrt{16 + 84}}{2} = \frac{-4 \pm \sqrt{100}}{2} = \frac{-4 \pm 10}{2} \Rightarrow x_1 = \frac{-4+10}{2} = 3$$

$$x_2 = \frac{-4-10}{2} = -7$$

6) $x - 8x + 12 = (x - 6)(x - 2)$

$a = 1, b = -8, c = 12$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4.1.12}}{2.1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2} \Rightarrow x_1 = \frac{8+4}{2} = 6 \text{ et } x_2 = \frac{8-4}{2} = 2$$

7) $x^2 + 13x - 14 = (x - 14)(x + 1)$

$a = 1, b = 13, c = -14$

$$x = \frac{13 \pm \sqrt{13^2 - 4.1.(-14)}}{2.1} = \frac{13 \pm \sqrt{169 + 56}}{2} = \frac{13 \pm \sqrt{225}}{2} = \frac{13 \pm 15}{2} \Rightarrow x_1 = \frac{13+15}{2} = 14$$

$$x_2 = \frac{13-15}{2} = -1$$

8) $2x^2 - 14x + 24 = 2(x - 4)(x - 3)$

$a = 2, b = -14, c = 24$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4.2.24}}{2.2} = \frac{14 \pm \sqrt{196 - 192}}{4} = \frac{14 \pm \sqrt{4}}{4} = \frac{14 \pm 2}{4} \Rightarrow x_1 = \frac{14+2}{4} = 4$$

$$x_2 = \frac{14-2}{4} = 3$$

9) $3x^2 + 3x - 6 = 3(x - 1)(x + 2)$

$a = 3, b = 3, c = -6$

$$x = \frac{-3 \pm \sqrt{3^2 - 4.3.(-6)}}{2.3} = \frac{-3 \pm \sqrt{9 + 72}}{6} = \frac{-3 \pm \sqrt{81}}{6} = \frac{-3 \pm 9}{6} \Rightarrow x_1 = \frac{-3+9}{6} = 1 \text{ et } x_2 = \frac{-3-9}{6} = -2$$

10) $3x^2 - 3x - 6 = (x - 6)(x + 1)$

$a = 3, b = -3, c = -6$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4.3.(-6)}}{2.3} = \frac{3 \pm \sqrt{9 + 72}}{6} = \frac{3 \pm \sqrt{81}}{6} = \frac{3 \pm 9}{6} \Rightarrow x_1 = \frac{3+9}{6} = 2 \text{ et } x_2 = \frac{3-9}{6} = -1$$

6. Exercices récapitulatifs : factorise (avec la méthode de ton choix)

$$1) 9x^2 + 2x + \frac{1}{9} = (3x + \frac{1}{3})^2$$

$$2) 2x^6 + 2 - 4x^3 = 2(x^6 - 2x^3 + 1) = 2(x^3 - 1)^2$$

$$3) x^4 - 2x^3 - x + 2 = (x^4 - x) - (2x^3 - 2) = x(x^3 - 1) - 2(x^3 - 1) = (x^3 - 1)(x - 2)$$

$$4) 3(x - y) - x(y - x) = 3(x - y) + x(x - y) = (x - y)(3 + x)$$

$$5) y^5 - \frac{2y^3}{3} + \frac{y}{9} = y(y^4 - \frac{2y^2}{3} + \frac{1}{9}) = y(y^2 - \frac{1}{3})^2$$

$$6) 81x^4 - 144 = 9(9x^4 - 16) = 9(3x^2 - 4)(3x^2 + 4) = 9(\sqrt{3}x - 2)(\sqrt{3}x + 2)(3x^2 + 4)$$

$$7) (2x + 1)^2 - (x + 1)^2 = [(2x + 1) - (x + 1)][(2x + 1) + (x + 1)] \\ = (2x + 1 - x - 1)(2x + 1 + x + 1) = x(3x + 2)$$

$$8) 5t^4 - 10t^2 + 5 = 5(t^4 - 2t^2 + 1) = 5(t^2 - 1)^2 = 5[(t - 1)(t + 1)]^2 = 5(t - 1)^2(t + 1)^2$$

$$9) 121a^2 - (2a + 3)^2 = [11a - (2a + 3)][11a + (2a + 3)] = (11a - 2a - 3)(11a + 2a + 3) \\ = (9a - 3)(13a + 3) = 3(3a - 1)(13a + 3)$$

$$10) x^2 + 4y^2 - 1 - 4xy = (x^2 - 4xy + 4y^2) - 1 = (x - 2y)^2 - 1 = [(x - 2y) - 1][(x - 2y) + 1] \\ = (x - 2y - 1)(x - 2y + 1)$$

$$11) u^3 - 6u^2 - u + 6 = (u^3 - 6u^2) + (-u + 6) = u^2(u - 6) - (u - 6) = (u - 6)(u^2 - 1) \\ = (u - 6)(u - 1)(u + 1)$$

$$12) -u^2 + 2u - 1 = -(u^2 - 2u + 1) = -(u - 1)^2$$

$$13) 8a^3 - 8a^2 + 2a = 2a(4a^2 - 4a + 1) = 2a(2a - 1)^2$$

$$14) a^3 - 3a^2 - a + 3 = (a^3 - 3a^2) + (-a + 3) = a^2(a - 3) - (a - 3) = (a - 3)(a^2 - 1) \\ = (a - 3)(a - 1)(a + 1)$$

$$15) x^2 - 12x - y^2 + 36 = (x^2 - 12x + 36) - y^2 = (x - 6)^2 - y^2 = (x - 6 - y)(x - 6 + y)$$

$$16) xy^2 - xz^2 = x(y^2 - z^2) = x(y - z)(y + z)$$

$$17) x^4y^2 - x^2y^4 = y^2x^2(x^2 - y^2) = y^2x^2(x - y)(x + y)$$

$$18) x^2y - 6xy + 9y = y(x^2 - 6x + 9) = y(x - 3)^2$$

$$19) 3x^3y - 3xy^3 = 3xy(x^2 - y^2) = 3xy(x - y)(x + y)$$

$$20) 0,04y^2 - 0,09x^2 = (0,2x - 0,3y)(0,2x + 0,3y)$$

$$21) x^8 - y^8 = (x^4 - y^4)(x^4 + y^4) = (x^2 - y^2)(x^2 + y^2)(x^4 + y^4) = (x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$$

$$22) x^2 - x + 4^{-1} = x^2 - x + 1/4 = (x - 1/2)^2 \text{ ou bien :} \\ = x^2 - x + 0,25 = (x - 0,5)^2$$

$$23) mn^4op^4 - mn^2op^2 = mn^2op^2(n^2p^2 - 1) = mn^2op^2(np - 1)(np + 1)$$

$$24) 2a^2 - 12a + 18 = 2(a^2 - 6a + 9) = 2(a - 3)^2$$

$$25) x^6 - \frac{1}{16} = (x^3 - 1/4)(x^3 + 1/4) \text{ ou bien :} \\ = (x^3 - 0,25)(x^3 + 0,25)$$

$$26) a^4 - 2a^2 + 1 = (a^2 - 1)^2 = [(a - 1)(a + 1)]^2 = (a - 1)^2(a + 1)^2$$

$$27) 5x^3 - 15xy^2 = 5x(x^2 - 3y^2) = 5x(x - \sqrt{3}y)(x + \sqrt{3}y)$$

$$28) a^4 - 6a^2 + 9 = (a^2 - 3)^2 = [(a - \sqrt{3})(a + \sqrt{3})]^2 = (a - \sqrt{3})^2(a + \sqrt{3})^2$$

$$29) 49x^3 - x = x(49x^2 - 1) = x(7x - 1)(7x + 1)$$

$$30) \frac{x^2}{4} + \frac{1}{49} - \frac{x}{7} = \left(\frac{x}{2} - \frac{1}{7}\right)^2$$

$$31) 9x^2y^2 - 16 = (3xy - 4)(3xy + 4)$$

$$32) 4x^2 - 2x + 1 = (2x - 1)^2$$

$$33) 4a^2b^3 - b = b(4a^2b^2 - 1) = b(2ab - 1)(2ab + 1)$$

$$34) 75a^2 + 30a + 3 = 3(25a^2 + 10a + 1) = 3(5a + 1)^2$$

$$35) (x + \sqrt{3})^2 - 16 = (x + \sqrt{3} - 4)(x + \sqrt{3} + 4)$$

$$36) 4x^2(x + 3) - (3 + x) = (x + 3)(4x^2 - 1) = (x + 3)(2x - 1)(2x + 1)$$

$$37) 3x^3y^2 - 5x^5 = x^3(3y^2 - 5x^2) = x^3(\sqrt{3}y - \sqrt{5}x)(\sqrt{3}y + \sqrt{5}x)$$

$$38) 9(x^2 - 3) - y^4(x^2 - 3) = (x^2 - 3)(9 - y^2) = (x - \sqrt{3})(x + \sqrt{3})(3 - y)(3 + y)$$

$$39) 3x(y^2 - 2y) + 3x = 3x[(y^2 - 2y) + 1] = 3x(y^2 - 2y + 1) = 3x(y - 1)^2$$

$$40) x^4 - 2x^2 + 1 = (x^2 - 1)^2 = [(x - 1)(x + 1)]^2 = (x - 1)^2(x + 1)^2$$

$$\begin{aligned} 41) (4x^2 + 12x)^2 + 9(4x^2 + 12x) &= (4x^2 + 12x)^2 [(4x^2 + 12x) + 9] = (4x^2 + 12x)^2 (4x^2 + 12x + 9) \\ &= [4x(x + 3)]^2 (2x + 3)^2 = (4x)^2 (x + 3)^2 (2x + 3)^2 \\ &= 16x^2 (x + 3)^2 (2x + 3)^2 \end{aligned}$$

$$\begin{aligned} 42) 3x^4(x^2 - \frac{4}{3}) - 75(x^2 - \frac{4}{3}) &= 3(x^2 - \frac{4}{3})(3x^4 - 25) \\ &= 3(x - \sqrt{\frac{4}{3}})(x + \sqrt{\frac{4}{3}})(\sqrt{3}x^2 - 5)(\sqrt{3}x^2 + 5) \\ &= 3(x - 2\sqrt{\frac{1}{3}})(x + 2\sqrt{\frac{1}{3}})(\sqrt{3}x^2 - 5)(\sqrt{3}x^2 + 5) \end{aligned}$$



7. Exercices supplémentaires (plus difficiles)

$$1) 2a(x+y) - 3b(-x-y) = 2a(x+y) + 3b(x+y) = (x+y)(2a+3b)$$

$$2) x(2a-b) + y(b-2a) = x(2a-b) - y(2a-b) = (2a-b)(x-y)$$

$$3) a(x-y) - (y-x) = a(x-y) + (x-y) = (x-y)(a+1)$$

$$4) (4a-2b)(2x-3y) + (3y-2x)(b-2a) = 2(2a-b)(2x-3y) + (2x-3y)(2a-b) \\ = 3(2a-b)(2x-3y)$$

$$5) a^2(x-1)(a+b) + a^3(1-x) = a^2[(x-1)(a+b) - a(x-1)] = a^2(x-1)[(a+b)-a] \\ = a^2(x-1)b = a^2b(x-1)$$

$$6) (x-2y)(a-b) - (b-a)(2x+y) = (x-2y)(a-b) + (a-b)(2x+y) \\ = (a-b)(x-2y+2x+y) = (a-b)(3x-y)$$

$$7) 5x^2 - 5 = 5(x^2 - 1) = 5(x-1)(x+1)$$

$$8) a^4x^4 - a^4 = a^4(x^4 - 1) = a^4(x^2 - 1)(x^2 + 1) = a^4(x-1)(x+1)(x^2 + 1)$$

$$9) a^4b^2x^5 - a^2x = a^2x(a^2b^2x^4 - 1) = a^2x(abx^2 - 1)(abx^2 + 1)$$

$$10) x^5y^7 - xy = xy(x^4y^6 - 1) = xy(x^2y^3 - 1)(x^2y^3 + 1) \\ = xy(\sqrt{x^2y^3} - 1)(\sqrt{x^2y^3} + 1)(x^2y^3 + 1) \\ = xy(xy\sqrt{y} - 1)(xy\sqrt{y} + 1)(x^2y^3 + 1)$$

$$11) (x+2)^3 - 3(x+2) = (x+2)((x+2)^2 - 3) = (x+2)((x+2) - \sqrt{3})(x+2) + \sqrt{3}) \\ = (x+2)(x+2 - \sqrt{3})(x+2 + \sqrt{3})$$

$$12) (a-b) - (a-b)x^4 = (a-b)(1-x^4) = (a-b)(1-x^2)(1+x^2) \\ = (a-b)(1-x)(1+x)(1+x^2)$$

$$13) -3x^9 + 3x^5 = 3x^5(-1+x^4) = 3x^5(x^2-1) = 3x^5(x-1)(x+1)$$

$$14) 2x^3 - 7x = x(2x^2 - 7) = x(\sqrt{2x} - \sqrt{7})(\sqrt{2x} + \sqrt{7})$$

$$15) 16a^4b^2 - 24a^2b^3 + 9b^4 = b^2(16a^4 - 24a^2b + 9b^2) = b^2(4a^2 - 3b)^2 = b^2(2a - \sqrt{3b})(2a + \sqrt{3b})$$

$$16) x^2(a^2 - 4) - (a^2 - 4) = (a^2 - 4)(x^2 - 1) = (a - 2)(a + 2)(x - 1)(x + 1)$$

$$17) (a + 1)^4 - (a + 1)^2 = (a + 1)^2 [(a + 1)^2 - 1] = (a + 1)^2 [(a + 1) - 1][(a + 1) + 1] \\ = (a + 1)^2 a (a + 2) = a (a + 1)^2 (a + 2)$$

$$18) 125x^3(x - y)^2 - 45x(3x + 2y)^2 = 5x [25(x - y)^2 - 9(3x + 2y)^2] = 5x [5^2(x - y)^2 - 3^2(3x + 2y)^2] \\ = 5x [(5(x - y))^2 - (3(3x + 2y))^2] \\ = 5x [5(x - y) - 3(3x + 2y)][5(x - y) + 3(3x + 2y)] \\ = 5x (5x - 5y - 9x - 6y)(5x - 5y + 9x + 6y) \\ = 5x (-4x - 11y)(14x - y) = -5x(4x + 11y)(14x - y)$$

$$19) (a^2 + b^2 - c^2)^2 - (a^2 - b^2 + c^2)^2 = [(a^2 + b^2 - c^2) - (a^2 - b^2 + c^2)][(a^2 + b^2 - c^2) + (a^2 - b^2 + c^2)] \\ = (a^2 + b^2 - c^2 - a^2 + b^2 - c^2)(a^2 + b^2 - c^2 + a^2 - b^2 + c^2) \\ = (2b^2 - 2c^2)2a^2 = 2(b^2 - c^2)2a^2 = 4a^2(b^2 - c^2) = 4a^2(b - c)(b + c)$$

$$20) \frac{(a - 1)^2}{3} - \frac{a^2}{12} = \frac{1}{3} [(a - 1)^2 - \frac{a^2}{4}] = \frac{1}{3} [(a - 1)^2 - (\frac{a}{2})^2] = \frac{1}{3} (a - 1 - \frac{a}{2})(a - 1 + \frac{a}{2}) \\ = \frac{1}{3} (\frac{2a - 2 - a}{2})(\frac{2a - 2 + a}{2}) = \frac{1}{3} (\frac{a - 2}{2})(\frac{3a + 2}{2})$$

$$21) x^2 - 8x + 12 = (x - 6)(x - 2)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 1, b = -8, c = 12$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2} \Rightarrow x_1 = \frac{8+4}{2} = \frac{12}{2} = 6 \text{ et } x_2 = \frac{8-4}{2} = \frac{4}{2} = 2$$

$$22) x^2 - 14x + 13 = (x - 13)(x - 1)$$

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4 \cdot 1 \cdot 13}}{2 \cdot 1} = \frac{14 \pm \sqrt{196 - 52}}{2} = \frac{14 \pm \sqrt{144}}{2} = \frac{14 \pm 12}{2} \Rightarrow x_1 = \frac{14+12}{2} = 13$$

$$x_2 = \frac{14-12}{2} = 1$$

$$23) x^2 - 22x + 85 = (x - 17)(x - 5)$$

$$x = \frac{22 \pm \sqrt{(-22)^2 - 4 \cdot 1 \cdot 85}}{2 \cdot 1} = \frac{22 \pm \sqrt{484 - 340}}{2} = \frac{22 \pm \sqrt{144}}{2} = \frac{22 \pm 12}{2} \Rightarrow x_1 = \frac{22+12}{2} = \frac{34}{2} = 17$$

$$x_2 = \frac{22-12}{2} = \frac{10}{2} = 5$$

24) $x^2 - 4x - 5 = (x - 5)(x + 1)$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4.1.(-5)}}{2.1} = \frac{4 \pm \sqrt{16+20}}{2} = \frac{22 \pm \sqrt{36}}{2} = \frac{4 \pm 6}{2} \Rightarrow x_1 = \frac{4+6}{2} = \frac{10}{2} = 5 \text{ et}$$
$$x_2 = \frac{4-6}{2} = \frac{-2}{2} = -1$$

25) $x^2 + 10x + 16 = (x + 2)(x + 8)$

$$x = \frac{-10 \pm \sqrt{10^2 - 4.1.16}}{2.1} = \frac{-10 \pm \sqrt{100 - 64}}{2} = \frac{-10 \pm \sqrt{36}}{2} = \frac{-10 \pm 6}{2} \Rightarrow x_1 = \frac{-10+6}{2} = -2$$
$$x_2 = \frac{-10-6}{2} = -8$$

26) $x^2 - 115x + 1500 = (x - 100)(x - 15)$

$$x = \frac{115 \pm \sqrt{(-115)^2 - 4.1.1500}}{2.1} = \frac{115 \pm \sqrt{13225 - 6000}}{2} = \frac{115 \pm \sqrt{7225}}{2} = \frac{115 \pm 85}{2} \Rightarrow x_1 = \frac{200}{2} = 100$$
$$x_2 = \frac{30}{2} = 15$$

27) $x^2 - 4x - 32 = (x - 8)(x + 4)$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4.1.(-32)}}{2.1} = \frac{4 \pm \sqrt{16 + 128}}{2} = \frac{4 \pm \sqrt{144}}{2} = \frac{4 \pm 12}{2} \Rightarrow x_1 = \frac{16}{2} = 8 \text{ et } x_2 = \frac{-8}{2} = -4$$

28) $x^2 + 5x - 14 = (x - 2)(x + 7)$

$$x = \frac{-5 \pm \sqrt{5^2 - 4.1.(-14)}}{2.1} = \frac{-5 \pm \sqrt{25 + 56}}{2} = \frac{-5 \pm \sqrt{81}}{2} = \frac{-5 \pm 9}{2} \Rightarrow x_1 = \frac{4}{2} = 2 \text{ et } x_2 = \frac{-14}{2} = -7$$

29) $x^2 + 20x + 19 = (x - 19)(x + 1)$

$$x = \frac{20 \pm \sqrt{20^2 - 4.1.19}}{2.1} = \frac{20 \pm \sqrt{400 - 76}}{2} = \frac{20 \pm \sqrt{324}}{2} = \frac{20 \pm 18}{2} \Rightarrow x_1 = \frac{38}{2} = 19 \text{ et } x_2 = \frac{-2}{2} = -1$$

30) $x^2 - 4x - 12 = (x - 6)(x + 2)$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4.1.(-12)}}{2.1} = \frac{4 \pm \sqrt{16 + 48}}{2} = \frac{4 \pm \sqrt{64}}{2} = \frac{4 \pm 8}{2} \Rightarrow x_1 = \frac{12}{2} = 6 \text{ et } x_2 = \frac{-4}{2} = -2$$



8. Autres exercices

$$1) \frac{2^{12}}{12^2} = \frac{2^{12}}{(2 \cdot 2 \cdot 3)^2} = \frac{2^{12}}{(2^2 \cdot 3)^2} = \frac{2^{12-4}}{2^4 \cdot 3^2} = \frac{2^8}{3^2} \quad \text{a) } \frac{2^8}{3^2} \quad \text{b) } \frac{2^{10}}{2} \quad \text{c) } \frac{2^6}{6} \quad \text{d) } \frac{1}{6} \quad \text{e) } 1$$

2) Si on écrit $3^5 \cdot 27^2 \cdot 3^2 \cdot 243$ sous la forme d'une puissance de 3, l'exposant sera égal à :

a) 10 b) 18 c) 21 d) 30 e) 450

$$3^5 \cdot 27^2 \cdot 3^2 \cdot 243 = 3^{5+2} \cdot (3 \cdot 9)^2 \cdot 243 = 3^7 \cdot (3 \cdot 3 \cdot 3)^2 \cdot (81 \cdot 3) = 3^7 \cdot (3^3)^2 \cdot (9^2 \cdot 3) \\ = 3^7 \cdot 3^{3 \cdot 2} \cdot ((3^2)^2 \cdot 3) = 3^7 \cdot 3^6 \cdot 3^4 \cdot 3 = 3^{7+6+4+1} = 3^{18}$$

$$3) \quad 6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6 = \quad \text{a) } 6^9 \quad \text{b) } 6^7 \quad \text{c) } 36^6 \quad \text{d) } 6^6 \quad \text{e) } 36^{36}$$

$$6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6 = 6 \cdot 6^6 = 6^{1+6} = 6^7$$

$$4) \quad \frac{2^{13} + 2^{14} + 2^{15}}{14} = \quad \text{a) } 2^{28} \quad \text{b) } 2^{14} \quad \text{c) } 2^{12} \quad \text{d) } 2^{10} \quad \text{e) } 2^6$$

$$\frac{2^{13} + 2^{14} + 2^{15}}{14} = \frac{2^{13} \cdot (1 + 2 + 2^2)}{2 \cdot 7} = \frac{2^{12} \cdot (1 + 2 + 4)}{7} = \frac{2^{12} \cdot 7}{7} = 2^{12}$$

